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3 (Sem-6/CBCS) MAT HE 4

2025

**MATHEMATICS**

(Honours Elective)

Paper : MAT-HE-6046

**(Hydromechanics)**

Full Marks : 80

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer the following questions:  $1 \times 10 = 10$
- (i) Define specific heat of a body.
  - (ii) Define surface of equal pressure.
  - (iii) For an irrotational flow  $\text{curl } \bar{q} \neq 0$ .  
(State True or False)
  - (iv) State Charle's law.
  - (v) Name the instrument used to measure atmospheric pressure.
  - (vi) What is tube of flow ?

- (vii) What is the physical meaning of  $\text{curl } \vec{q}$ ?
- (viii) Define absolute zero of temperature.
- (ix) Name the *two* methods of treating the general problem of Hydrodynamics.
- (x) What is meant by rotational flow ?

2. Answer the following questions :  $2 \times 5 = 10$

(a) Show that the surface of equal pressure is intersected orthogonally by the lines of force.

(b) If  $\rho_0$  and  $\rho$  be the densities of a gas at  $0^\circ$  and  $t^\circ$  centigrade respectively, then establish the relation  $\rho_0 = \rho(1 + \alpha t)$

where  $\alpha = \frac{1}{273}$ .

(c) Establish the relation between local and individual time rate of change.

(d) What is meant by velocity potential? Does velocity potential exist for a rotational flow ?

(e) Define streamline and path line of a fluid particle.

3. Answer **any four** of the following questions :  
5×4=20

(a) A mass of fluid is at rest under the forces

$$X = (y + z)^2 - x^2, Y = (z + x)^2 - y^2,$$

$Z = (x + y)^2 - z^2$ ; find the density and the surfaces of equal pressure.

(b) A fluid at rest is in equilibrium in the

$$\text{forces field } X = y^2 + z^2 - xy - zx,$$

$$Y = z^2 + x^2 - yz - yx,$$

$Z = x^2 + y^2 - zx - zy$ . Show that the curves of equal pressure and density are a set of circles.

(c) If the absolute temperature  $T$  at a height  $z$  is a function of the pressures at two heights  $z_1$  and  $z_2$  then show that

$$\log \frac{p_2}{p_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{f(z)}$$

$R$  being the constant in the equation  
 $p = R\rho T$ .

(d) The velocity components of a flow in cylindrical polar coordinates are  $(r^2 z \cos \theta, r z \sin \theta, z^2 t)$ . Determine the components of the acceleration of a fluid particle.

(e) Test whether the motion specified by

$$\vec{q} = \frac{k^2 (xi - yj)}{x^2 + y^2}, \quad k \text{ is constant, is a}$$

possible motion for an incompressible fluid. If so, determine the equations of streamlines.

(f) The particles of a fluid motion move symmetrically in space with regard to a fixed centre; prove that the equation of

$$\text{continuity is } \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \frac{\rho}{r^2} \frac{\partial}{\partial r} (ur^2) = 0,$$

where  $u$  is the velocity at a distance  $r$ .

4. Answer **any four** parts : 10×4=40

(a) (i) Show that  $\frac{x^2}{a^2} \tan^2 t + \frac{y^2}{b^2} \cot^2 t = 1$  is

a possible form of bounding surface of a liquid.

- (ii) Two volumes  $V_1$  and  $V_2$  of different gases at pressures  $P_1$  and  $P_2$  and absolute temperatures  $T_1$  and  $T_2$  are mixed together, so that the volume of the mixture is  $V$  and absolute temperature is  $T$ . Prove that the pressure of the mixture is

$$\frac{T}{V} \left( \frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2} \right).$$

- (b) (i) A mass of homogeneous liquid, contained in a vessel, revolves uniformly about a vertical axis. Determine the pressure at any point and the surfaces of equal pressure.

- (ii) Show that the centre of pressure of a circular area immersed in the liquid whose centre is at a depth  $h$  below the surface, when the density of the liquid varies as the depth, is

at a depth  $\frac{2a^2 h}{a^2 + 4h^2}$  below the centre of the circle, where  $a$  is the radius of the circular area.

- (c) (i) A liquid of given volume  $V$  is at rest under the forces

$$X = \frac{\mu x}{a^2}, Y = -\frac{\mu y}{b^2}, Z = -\frac{\mu z}{c^2}. \text{ Find}$$

the pressure at any point of the liquid and the surfaces of equal pressure.

- (ii) For a perfect gas, establish the relation  $C_p - C_v = R$ , where the symbols have their usual meanings.

- (d) Obtain Euler's equation of motion for a non-viscous fluid in the form

$$\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho} \vec{\nabla} P.$$

- (e) Obtain the equation of continuity for a fluid in motion in the form

$$\frac{\partial p}{\partial t} + \text{div}(P\vec{q}) = 0 \text{ where } p \text{ and } \vec{q} \text{ are}$$

respectively the density and the velocity of the fluid. Deduce the form of equation of continuity when the fluid is homogeneous and incompressible.

- (f) Stream is rushing from a boiler through a conical pipe, the diameters of the ends of which are  $D$  and  $d$ ; if  $V$  and  $v$  be the corresponding velocities of the stream and if the motion be supposed to be that of divergence from the vertex of the cone,

prove that  $\frac{v}{V} = \frac{D^2}{d^2} e^{\frac{v^2 - V^2}{2k}}$  where  $k$  is

the pressure divided by the density and suppose constant.

- (g) Prove that if the forces per unit of mass at  $(x, y, z)$  parallel to the axes are  $y(a - z), x(a - z), xy$ ; the surface of equal pressure are hyperbolic paraboloid and the curves of equal pressure and density are rectangular hyperbolas.

- (h) A hemispherical bowl is filled with water and two vertical planes are drawn through its central radius, cutting off a semilune of the surface, If  $2\alpha$  be the angle between the planes, prove that the angle which the resultant pressure on the surface makes with the vertical is

$$\tan^{-1} \left( \frac{\sin \alpha}{\alpha} \right)$$