

2026
(Regular)
Physics (Core)
Paper Code : PY-CE-4214

Paper Name : Mathematical Physics - IV

Time : 2 hours

Full Marks : 45

The figures in the margin indicate full marks for the questions.

1. Choose the correct option :

1×4=4

a) The generating function for the Legendre polynomial is :

i) $(1 - 2xz + z^2)^{-1}$

ii) $(1 - 2xz + z^2)^{-\frac{1}{2}}$

iii) $(1 - 2xz - z^2)^{-\frac{1}{2}}$

iv) $(1 + 2xz - z^2)^{\frac{1}{2}}$

b) The value of δ_{ii} in n- space dimensions is :

i) 3

ii) d

iii) 1

iv) n

c) What is the fundamental period of the function

$$f(x) = \frac{a_0}{2} + \sum (a_n \cos nx + b_n \sin nx) ?$$

i) π

ii) 2π

iii) 4π

iv) None of the above

d) The error in Simson's $\frac{1}{3}$ rule is proportional to :

i) h

ii) h^2

iii) h^3

iv) h^4

Contd..

2. Answer the following questions : 2×3=6

- a) What is the principle behind the Gauss elimination method ?
- b) Write down the expressions for Fourier Coefficients.
- c) Write down the tensorial expressions for $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$.

3. Answer **any three** questions from the following : 5×3=15

- a) Solve the following system using Gauss elimination :
 $2x + y + z = 5$, $4x - 6y = -2$, $-2x + 7y + 2z = 9$.

- b) Evaluate : $\int_{-\pi}^{\pi} \sin^2 x \, dx$ using orthogonality relations.

- c) Show that $\delta_{ij} A_j = A_i$ and interpret the result.

- d) Apply the bisection method to find a root of $x^3 - x - 1 = 0$ correct up to two iterations.

- e) Derive Simpson's $\frac{1}{3}$ rule by approximating the integral $\int_a^b f(x) \, dx$ by a quadratic polynomial passing through three equally spaced points.

- f) Expand $f(x) = x^2$ in Fourier series in the interval $(-\pi, \pi)$.

4. Answer **any two** questions from the following : 10×2=20

- a) i) State the conditions under which Frobenius method is applicable to differential equations. 4

- ii) Solve the differential equation $x^2 y'' + xy' - y = 0$ using Frobenius method and obtain the series solution. 6

- b) Using the recurrence relation for Legendre polynomial $P_n(x)$,

show that $\int_{-1}^1 x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1}$. Express associate

Legendre polynomial $P_n^m(x)$ in terms of Legendre polynomial $P_n(x)$. 5+5=10

- c) Write down the Hermite differential equation. Write down its generating function and show that $H'_n(x) = 2n H_{n-1}(x)$.

2+2+6=10

- d) Express any arbitrary tensor T_{ij} in terms of its symmetric and antisymmetric parts. Show that the product of a symmetric and antisymmetric tensor is zero.

Write down the transformation laws for a covariant and a contravariant tensor. 2+4+4=10

