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FYUGP/Sem-6/ST-CE3

2026

Statistics (Core)

Paper Code : ST-CE-6334

Paper Name : Statistical Inference - II

Time : 2 hours *Full Marks : 50*

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×4=4

   a) The level of significance α of a test is also known as _____ of the critical region. (Fill in the blank)

   b) Neyman-Pearson test gives us the best critical region. (State true or false)

   c) Power of the test is greater than or equal to :

   i) size of the critical region

   ii) (size of the critical region) - 1

   iii) (size of the critical region) + 1

   iv) none of the above

   (Choose the correct option)

   d) The choice of one tailed or two tailed test depends on the _____.
   (Fill in the blank)

2. Answer the following questions briefly : 2×3=6

   a) "Let $x_1, x_2, x_3, ... x_n$ be independent and identically distributed (IID) normal variables with common mean μ and common variance σ^2, then the hypothesis $H_0 : \mu = 10$ is a composite

Contd.

hypothesis". State whether the above statement is true or false, giving reasons in support of your answer.

b) What is a power curve ?

c) Define Uniformly most powerful test.

3. Answer *any two* of the following questions : 5×2=10

a) Write the steps in solving a testing a hypothesis problem. 5

b) i) Define Simple hypothesis and Composite hypothesis. 2

ii) Write briefly on Sequential statistical inference. 3

c) Let p be the probability that a coin will fall in head in a single toss in order to test $H_0 : p = 1/2$ against $H_1 : p = 3/4$. The coin is tossed 5 times and H_0 is rejected, if more than 3 heads are obtained. Find the probability of type-I error and the power of the test. 5

d) Explain the SPRT to test the hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta = \theta_1$ with p.d.f. $f(x, \theta)$. 5

4. Answer *any three* of the following questions : 10×3=30

a) i) Explain briefly the concept of most powerful test. 3

ii) Given the frequency function

$$f(x, \theta) = \frac{1}{\theta}, 0 \le x \le \theta$$

and that you are testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$ by means of a single observed value of x. What would be the probabilities of type I and type II errors if you choose the intervals

A) $0.5 \le x$ and B) $1 \le x \le 1.5$

as the critical region ? Also obtain the power functions of the test. 7

b) i) State and prove the Neyman Pearson lemma. 2+5=7

ii) Explain briefly the Likelihood ratio test. 3

c) i) Given a random sample $x_1, x_2, x_3, ... x_n$ from a distribution with pdf

$$f(x,\theta) = \theta e^{-\theta x}, x > 0$$

Show that there exists no UMP test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. 6

ii) State the properties of Sequential probability ratio test. 4

d) i) If W be a Most powerful critical region of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, then prove that it is necessarily unbiased. 5

ii) Examine whether a best critical region exists for testing the null hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ for the parameter θ of the distribution

$$f(x,\theta) = \frac{1+\theta}{(x+\theta)^2}, 1 \leq x < \infty.$$ 5

e) Use the Neyman-Pearson lemma to obtain the best critical region for testing $\theta = \theta_0$ against $\theta = \theta_1 > \theta_0$ and $\theta = \theta_1 < \theta_0$ in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. Hence find the power of the test. 10

f) i) If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1:\theta=1$, on the basis of a single observation from the population

$f(x,\theta)=\theta e^{-\theta x}, 0\leq x<\infty$.

Obtain the values of type I and type II errors. 6

ii) Explain the terms : Critical region and errors of two kinds. 2+2=4

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