FYUGP/Sem-6/ST-CE4

2026

Statistics (Core)

Paper Code : ST-CE-6344

Paper Name : Multivariate Analysis and Non-parametric Test

Time : 2 hours *Full Marks : 50*

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×4=4

a) What is dispersion matrix in Multivariate data analysis ?

b) Let *(X, Y)* follows bivariate normal distribution with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ Then $E(X)=$_____. (Fill in the blank)

c) If the marginal distributions of *X* and *Y* are normal, it does not necessarily imply that the joint distribution of *(X, Y)* is bivariate normal. (State true or false)

d) Ordinary sign test considers the difference of observed values from the hypothetical median value in terms of

i) Sign only

ii) Mangnitude only

iii) Sign and Mangnitude only

iv) None of the above (Choose the correct option)

2. Answer the following questions : 2×3=6

a) State the probability mass function of multinomial distribution with parameters $(n, k, p_1, p_2, ..., p_k)$, defining the parameters.

b) Find $\text{var}(A\underset{\sim}{X})$, where A is a $p \times p$ matrix of constant elements and X is a p×1 vector with variance-covariance matrix Σ.

c) Define empirical distribution function.

Contd..

3. Answser the following questions *(any two)* : 5×2=10

a) Obtain the probability generating function of multinomial distribution with parameters $(n, k, p_1, p_2, \ldots\ldots p_k)$

b) Let *(X, Y)* follows BVN $(0,0,1,1,\rho)$. Then show that $X+Y$ and X-Y are independently distributed.

c) What is run ? Describe the test for randomness for a single sample based on median. 2+3=5

d) Define random vector, mean vector and correlation matrix. If X follows a p variate *(p>2)* distribution with $E(X)=\mu$ and $Var(X)=\Sigma$, then show that

$E(XX')=\Sigma+\mu\mu'$ 3+2=5

4. Answer of the following questions *(any three)* : 10×3=30

a) i) Derive the moment generating function $M_X(t)$ of the bivariate normal distribution (with usual parameters). 7

ii) Differentiate between parametric and non-parametric test. 3

b) i) Examine whether Hotelling T² is invariant under changes in the units of measurements. 5

ii) Describe one sample sign test for testing the null hypothesis that the population median is a given value. 5

c) i) Let $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$. Consider the tranformation $\underset{\sim}{X} - \underset{\sim}{\mu} = C\underset{\sim}{Y}$

where C is $p \times p$ non- singular matrix and $\underset{\sim}{Y} = (y_1\, y_2 \ldots y_p)'$. Prove that the corresponding Jacobian of transformation is $|J| = |\Sigma|^{1/2}$. 5

ii) Discuss the application of Hotelling T^2 based on two samples. 5

d) i) Let *(X, Y)* follows BVN $(0,0,1,1,\rho)$. Then find the conditional distribution of *Y* given *X*. 4

ii) Let *X* follows multinomial $(m,k,p_1,p_2,.....p_k)$ and *Y* follows multinomial $(n,k,p_1,p_2,...p_k)$. Also let X and Y are independent. Then show that $X+Y$ is multinomial $(m+n,k,p_1,p_2,....p_k)$. 6

e) i) Let $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu},\Sigma\right)$ Then show that $Q=(X-\mu)'\Sigma^{-1}(X-\mu)$ follows chi-square distribution with p-degrees of freedom. 6

ii) Describe briefly the Kruskal-Wallis non-parametric test. 4

f) i) Write an explanatory note on test of randomness. 5

ii) Let X_1 and X_2 be jointly normally random variables with parameter

$$\mu_1 = 2, \quad \sigma_1^2 = 1, \quad \mu_2 = 1, \quad \sigma_2^2 = 4, \quad \rho = \tfrac{1}{2}.$$

(A) Find $P(X_1 + 2X_2 \le 3)$

(B) Find $Cov(2X_1 + X_2, X_1 - X_2)$

(C) Find $E(X_2 / X_1 = 2)$ and $V(X_2 / X_1 = 2)$

(Given $P(Z \le 0.22) = 0.5821$) 2+1+2=5

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