FYUGP/Sem-4/ST-CE1

2026
(Regular)
Statistics (Core)
Paper Code : ST-CE-4214
Paper Name : Algebra

Time : 2 hours *Full Marks : 45*

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1\times4=4$

a) If α, β, γ be the roots of the equation,

$a_0x^3 + a_1x^2 + a_2x + a_3 = 0$, then sum of the roots are :

i) $-a_1/a_0$ ii) $-a_2/a_0$ iii) $-a_3/a_0$ iv) $-a_1/a_2$

(Choose the correct option)

b) Show that, $|A'| = |A|$. Where A is a square matrix of order n.

c) Every diagonal elements of Hermitian matrices are _____.

(Fill in the blank)

d) If A is a 4×3 matrix and its row-rank is 2, what is the maximum number of linearly independent columns ?

i) 2 ii) 3 iii) 4 iv) 1

(Choose the correct option)

2. Answer the following questions : $2\times3=6$

a) Solve the equation,

$x^3 - 5x^2 - 16x + 80 = 0$, the sum of two of its roots being equal to zero.

b) Show that the inverse of a non-singular matrix is unique.

Contd...

c) If $U = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, find the ranks of U and U^2.

3. Answer *any three* of the following questions : $5 \times 3 = 15$

a) Prove that every algebraic equation of degree n has only n roots.

b) Show that the vector $\bar{b} = \begin{pmatrix} -3 \\ 8 \\ 1 \end{pmatrix}$ is not in span $\{\bar{a}_1 \, \bar{a}_2\}$, where

$\bar{a}_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\bar{a}_2 = \begin{pmatrix} 5 \\ -13 \\ -3 \end{pmatrix}$.

c) Find the inverse of the matrix $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that

the transform of the matrix $A = \begin{bmatrix} b+c & c+a & b-a \\ c-b & c+b & a-b \\ b-c & a-c & a+b \end{bmatrix}$ by S.

i.e SAS^{-1} is a diagonal matrix.

d) Prove that - "The characteristic roots of a unitary matrix are of unit modulus".

e) If A be any square matrix, prove that $A+A^{\theta}, A A^{\theta}, A^{\theta} A$ are all Hermitian and $A-A^{\theta}$ is skew-Hermitian.

f) Suppose V is a vector space of dimension n. Let $S=\{v_1, v_2 \ldots v_m\}, m<n$, be a linearly independent subset of m vectors. Show that S can be extended to form a basis V.

4. Answer *any two* of the following questions : 10×2=20

a) i) If the roots of $x^3+3px^2+3qx+r=0$ are in harmonic progression, prove that, $2q^3=r(3pq-r)$. 5

ii) Find the condition that the roots of the equation $x^3+px^2+qx+r=0$ are in AP. 3

iii) State the fundamental theorem of Algebra. 2

b) Determine the characteristic roots and the corresponding characteristic vectors of the matrix, $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. 10

c) i) Define Circulant determinant and Vandermonde determinant for n^{th} order. 4

ii) Let P and Q be non-singular matrices. Show that if,

$A=\begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}$ then $A^{-1}=\begin{bmatrix} P^{-1} & 0 \\ 0 & Q^{-1} \end{bmatrix}$. 4

iii) Determine the eigenvalues of the matirx, $A=\begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & c & c \end{bmatrix}$. 2

d) i) Prove that the quadratic form

$6x_1^2+3x_2^2+14x_3^2+4x_2x_3+18x_3x_1+4x_1x_2$ in three variables is positive definite. 6

ii) Solve completely the system of equations : 4

$$x+y+z=0$$
$$2x-y-3z=0$$

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