2026

(Regular)

Statistics (Core)

Paper Code : ST-CE-4224

Paper Name : Theory of Probability Distributions

Time : 2 hours *Full Marks : 45*

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1\times4=4$

   a) The sum of independent beta variates is also a beta variate. (State true or false)

   b) Name a continuous distribution that possesses the 'memory less' property.

   c) If X is a Bernoulli random variable, find Var (X).

   d) If the cumulative distribution function of a continuous random variable x is $F(x)$, find the cumulative distribution function of $y=x+a$.

2. Answer the following questions : $2\times3=6$

   a) State the p.d.f. of Weibull distribution and the condition under which the Weibull p.d.f. reduces to exponential distribution.

   b) The distribution function F of a continuous x is given by

$$F(x)=\begin{cases} 0 & ,x<0 \\ x^2 & ,0\le x\le\frac{1}{2} \\ 1-\frac{3(3-x)^2}{25} & ,\frac{1}{2}\le x<3 \\ 1 & ,x\ge 3 \end{cases}$$

   Find the p.d.f of x with comments.

Contd...

c) Under what condition negative binomial distribution may be regarded as the generalization of geometric distribution and how ?

3. Answer *any three* of the following questions : 5×3=15

a) State the p.d.f of uniform distribution with parameters a and $b(a<b)$. If $X \sim \cup\left(\frac{\pi}{6},\frac{\pi}{2}\right)$ then find $P(\cos X > \sin X)$.

1+4=5

b) Derive the p.m.f. of negative binomial distribution and write down its mean and variance. 4+1=5

c) Show that hypergeometric distribution tends to binomial distribution under certain conditions.

d) If $E\left(X^2\right)=\sum k^2 p_k$ exists, then prove that

$$E\left(X^2\right)=p''(1)+p'(1)=2Q'(1)+Q(1) \text{ and hence}$$

$$V\left(X^2\right)=2Q'(1)+Q(1)-\{Q(1)\}^2=p''(1)+p'(1)-\{p'(1)\}^2$$

e) Let $X \sim N\left(0,\sigma^2\right)$ and $Y \sim N\left(0,\sigma^2\right)$ be two independent random variables. show that $\frac{X}{Y}$ follows a cauchy distribution.

f) Show that for a normal distribution with mean μ and variance σ^2 the central moments satisfy the relation

$$\mu_{2n}=\sigma^2(2n-1)\mu_{2n-2}.$$

4. Answer *any two* of the following questions : 10×2=20

a) Two random variables x and y have the joint probability density function :

$$P_{XY}(x,y)=\frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!}, \quad \begin{array}{l} y=0,1,2,.... \\ x=0,1,2,.... \end{array}$$

where λ, p are constants with $\lambda > 0$ and $0 < p < 1$.

Find :

i) The marginal probability density function of X and Y. 5

ii) The conditional distribution of Y for a given X and X for a given Y. 5

b) i) Define characteristic function and state its important properties. Also state the uniqueness theorem of characteristic function. 1+3+1=5

ii) Let $X \sim Exp(\lambda)$. Find its cumulative distribution function (cdf). Hence, obtain $P(X > 3)$ and $P(1 < X < 3)$. 3+1+1=5

c) i) Let (X, Y) be a two-dimensional non-negative continuous random variable having the joint density.

$$f(x,y)=\begin{cases}4xye^{-(x^2+y^2)} & ; x \geq 0,\ y \geq 0\\ 0 & ; otherwise\end{cases}$$

Prove that the density function $U = \sqrt{X^2 + Y^2}$ is -

$$h(u)=\begin{cases}2u^3e^{-u^2} & ;\ 0 \leq \cup < \infty\\ 0 & ;\ otherwise\end{cases}$$ 5

ii) Let $X \sim$ Beta (m,n) of 1st kind. If $m = 1$, $n = 1$ then find the pdf of X. 2

iii) Let $X \sim \cup(0,25)$. Find $P(X > 12 / X > 8)$. 3

d) i) Show that, with ususal notations,

$$V(X) = E[V(X|Y)] + V[E(X|Y)].$$ 5

ii) Define cumulant generating function. Show that

$Mean = k_1 \quad \mu_2 = k_2 \quad \mu_4 = k_3 \quad \mu_6 = k_4 + 3k_2^{\ 2}$. 5

(notations have their usual meaning)

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