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3 (Sem–3/CBCS) STA HC 3

2021

(Held in 2022)

STATISTICS

(Honours)

Paper : STA–HC–3036

(Mathematical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1 \times 7 = 7$

   (a) Find the infimum and supremum of the set $\left\{ \frac{(-1)^n}{n};\ n \in N \right\}$.

(b) Identify the wrong statement :

(i) The set R of real numbers is an open set.

(ii) The set of Q of rationals is an open set.

(iii) The set $\left\{\frac{1}{n} : n \in N\right\}$ is not open.

(c) Show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\ldots$ is not convergent.

(d) Give the interpretation of Rolle's theorem.

(e) Suppose Σu_n is a positive term series, such that

$$\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=l.$$

This series converges if

(i) $l > 1$

(ii) $l < 1$

(iii) $l = 1$

(iv) $l = 0$

(Choose the correct option)

(f) Which of the following is not correct?

(i) $\delta = E^{1/2} - E^{-1/2}$

(ii) $\Delta\nabla = \Delta - \nabla$

(iii) $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$

(iv) $\Delta^2 = E^2 + 2E + 1$

(g) Which of the following is not correct?

(i) Weddle's rule is more accurate than the Simpson's rule.

(ii) Weddle's rule requires at least seven consecutive values of y.

(iii) In Weddle's rule y is of the form
$$y = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$$

(iv) None of the above

2. Answer the following questions : 2×4=8

(a) Show that the sequence $\{S_n\}$, where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ is not convergent.

(b) If M and N are neighbourhood of a point x, then show that $M \cap N$ is also a neighbourhood of x.

(c) Show that $\sin x$ is uniformly continuous on $[0, \infty]$.

(d) State the properties of divided differences.

3. Answer **any three** of the following questions : 5×3=15

(a) Show that every convergent sequence is bounded and has a unique limit.

(b) Define positive term series. Show that the positive term geometric series $1+r+r^2+...$ converges for $r<1$ and diverges to $+\infty$ for $r \geq 2$.

(c) State and prove first mean value theorem of differential calculus.

(d) (i) Show that

$$\Delta x^m - \frac{1}{2}\Delta^2 x^m + \frac{1.3}{2.4}\Delta^3 x^m - \frac{1.3.5}{2.4.6}\Delta^4 x^m + \ldots m \text{ terms}$$

$$= \left(x+\frac{1}{2}\right)^m - \left(x-\frac{1}{2}\right)^m$$

(ii) Define Limit superior and Limit inferior.

(e) Prove that Newton-Gregory formula is a particular case of Newton's divided formula.

4. (a) (i) If $\lim_{n\to\infty} a_n = l$, then show that

$$\lim_{n\to\infty}\left(\frac{a_1+a_2+\ldots+a_n}{n}\right)=l$$ 8

(ii) Verify whether Rolle's theorem is applicable to the function $f(x)=2+(x-1)^{2/3}$ in the interval $[0, 2]$ or not. 2

Or

(b) (i) Show that the sequence $\{S_n\}$, where $S_n=\left(1+\frac{1}{n}\right)^n$ is convergent and that limit $\left(1+\frac{1}{n}\right)^n$ lies between 2 and 3.

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(ii) State Cauchy's nth root test. 2

5. (a) (i) State and prove Stirling interpolation formula. 7

(ii) Solve the difference equation

$$y_{k+1} - ay_k = 0, \; a \neq 1$$ 3

Or

(b) (i) Expand $\sin x$ by Maclaurin's infinite series. 8

(ii) State Taylor's theorem with Cauchy's form of remainder. 2

6. (a) (i) State and prove Weddle's rule. 7

(ii) Show that

$$\mu^2 y_x = y_x + \frac{1}{4}\delta^2 y_x$$ 3

Or

(b) (i) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} + \ldots + \frac{1}{\sqrt{2n}}\right] = \infty$$

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(ii) Define absolute convergence and conditional convergence.

Show that every absolutely convergent series is convergent. 6
